Logic for Computer Science - Exam - January 23rd, 2024

Name (use capital letters): .....

Group and Year: .....

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1. The domain is the set of reals. Translate the following proposition into first-order logic (step I: identify the predicates and the functions; step II: associate a signature; step III: write down the formula):

   No matter how we choose a prime number, there is a prime number greater than it.

2. The domain is the set of persons. Translate the following proposition into first-order logic (step I: identify the predicates and the functions; step II: associate a signature; step III: write down the formula):

   All students pay attention at the lecture, but not all students learn.

3. Show, by using a semantic argument, that:

the formula $((\exists x.P(x)) \vee \neg P(a))$ *is valid.*

4. *Define the function free (which computes the free variables in a formula).*

5. Find a formal proof using natural deduction for the following sequent:

$$\{(\exists x.P(x))\} \vdash (\exists x.(P(x) \vee Q(x))). \ .$$

Rough copy.